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Students' mathematical representation ability in Learning fractions through a concrete-representational-abstract approach with a joyful learning nuance

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Abstract

Mathematical representation ability is crucial in understanding fraction concepts, especially for elementary school students still in the concrete cognitive development stage. This study aims to describe the mathematical representation abilities of fourth-grade students in learning fractions through the Concrete-Representational-Abstract (CRA) approach with a joyful learning atmosphere. This approach was implemented through educational games such as Run Speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction. The study employed a descriptive qualitative method, involving three fourth-grade students from an elementary school in Cirebon, each representing a high, medium, and low ability level. Data were collected through written tests and in-depth interviews, and analysed using the Miles and Huberman model. The results indicate that when delivered in an engaging and enjoyable atmosphere, the CRA approach effectively supports the gradual development of verbal, visual, and symbolic representations. High-ability students demonstrated consistency across all three forms of representation; medium-ability students showed basic understanding but still required guidance; low-ability students faced difficulties yet remained enthusiastic throughout the learning process. This study contributes to developing instructional strategies that promote gradual conceptual understanding, foster enjoyable learning experiences, and encourage active student engagement in mathematics.

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1. Introduction

Mathematical representation ability is one of the essential competencies in mathematics learning, as it enables students to transform ideas or situations into mathematical forms such as symbols, diagrams, or verbal models. The National Council of Teachers of Mathematics (NCTM) emphasises that representation is not merely a means for solving problems, but also a fundamental mathematical thinking tool for developing deep conceptual understanding. Students can bridge their understanding between concrete and abstract concepts through representation and connect prior knowledge with new information. Mathematical representation also assists students in understanding relationships among ideas and in constructing more meaningful knowledge structures (Goldin, 2014).

Mathematical representation ability encompasses various forms of expressing mathematical ideas, such as verbal, visual, and symbolic representations, each plays a vital role in helping students build meaningful conceptual understanding. According to Mardiani et al. (2024), mathematical representation is the students' ability to interpret a mathematical problem into visual, symbolic, and linguistic forms. Similarly, Miranda et al. (2022) state that mathematical representation refers to the ability to express mathematical ideas or information in specific forms that support students in understanding, exploring, and solving mathematical problems more meaningfully. These representations are used in problem-solving and serve as tools in the learning process to develop connections between concepts and strengthen students' understanding.

In practice, based on preliminary observations conducted at an elementary school in Cirebon, it was found that many students still struggled to fully represent the concept of fractions. Students appeared confused when trying to relate the meaning between parts and the whole, and often made mistakes in distinguishing between the numerator and the denominator. Moreover, when asked to draw or explain fractions verbally, most students demonstrated inconsistent understanding and tended to guess. These findings are in line with the view of Faradiba et al. (2019), who stated that students often struggle to express mathematical concepts, particularly fractions, through visual and verbal representations due to their lack of comprehensive conceptual understanding. As a result, their representations tend to be less meaningful and do not accurately reflect their mastery of the material.

These difficulties indicate that mathematics instruction at the elementary level needs to be designed better to support the development of students' mathematical representation abilities. One strategy that can be implemented is a gradual approach that begins with concrete experiences and progresses toward abstract symbolic understanding. In this context, the Concrete-Representational-Abstract (CRA) approach emerges as a strategic alternative to address these challenges effectively. The CRA approach emphasises a systematic learning progression starting with using concrete objects, then visual representations, and eventually reaching abstract symbolisation. This approach helps students build conceptual understanding by providing structured and meaningful learning experiences.

The CRA approach can be integrated with the principles of joyful learning to create meaningful and enjoyable learning. Joyful learning emphasises engaging and enjoyable learning experiences that motivate students to participate actively in the learning process, according to Anggoro et al. (2017), a joyful and pleasant learning approach can create a more comfortable learning environment. Similarly, Tukarno (2012) explains that learning can be seen as a condition that evokes, expresses, or generates joy and happiness, allowing students to enjoy learning. Islahudin et al. (2025) proposed that joyful learning also helps reduce math anxiety and allows students to think creatively in constructing their own representations. This approach fosters students' cognitive development and considers the emotional and social aspects that support a more holistic learning process.

Several previous studies have investigated the effectiveness of the Concrete-Representational-Abstract (CRA) approach in enhancing students' mathematical representation abilities. Putri et al. (2020) found that students who participated in CRA-based instruction were better able to connect concrete concepts with symbolic representations, thereby strengthening their ability to solve mathematical problems. In addition, research by Yabo (2020) showed that using joyful learning approaches such as scaffolding through math games, puzzles, and hands-on activities can increase students' motivation and active participation during the learning process. This approach improves students' mathematical performance and reinforces their positive attitudes toward the subject. Furthermore, the study by Isnaeni et al. (2020) demonstrated that implementing the CRA approach, supported by manipulatives, significantly enhanced students' mathematical representation abilities, particularly in three-dimensional shapes with flat surfaces.

Research on the Concrete-Representational-Abstract (CRA) approach and joyful learning has been widely conducted; however, these two approaches are generally examined separately. Few studies have explicitly integrated the CRA approach with a joyful learning atmosphere, particularly in teaching fractions to fourth-grade elementary students. Yet, fractions are inherently abstract and often pose significant challenges for students at this level. This situation indicates a gap in the existing literature that warrants further investigation.

Therefore, this study contributes to developing effective mathematics teaching methods and efforts to create a positive and motivating learning environment. This research aims to describe the mathematical representation abilities of fourth-grade students in learning fractions through the Concrete-Representational-Abstract (CRA) approach infused with joyful learning in elementary schools in Cirebon. Based on this aim, the research seeks to answer the following questions:

1. How do fourth-grade students represent fraction concepts verbally, visually, and symbolically after learning through the CRA approach with a joyful learning nuance?
2. What are the characteristics of mathematical representations among students with high, moderate, and low abilities in understanding fractions using this approach?

2. Literature Review

2.1 Concrete-Representational-Abstract (CRA) Approach

The Concrete–Representational–Abstract (CRA) approach is a structured mathematics instructional strategy that supports students in gradually developing conceptual understanding through three stages: the use of concrete manipulatives (concrete), visual representations (representational), and abstract symbols (abstract). According to AL-Salahat (2022), CRA is effective in strengthening students' understanding of abstract concepts such as fractions and numerical operations.

The CRA approach is based on the principle that strong mathematical understanding is built through concrete experiences before students are introduced to abstract symbols. In its implementation, teachers guide students first to grasp concepts using real objects, then present visual representations such as pictures or diagrams, and finally progress toward using formal mathematical symbols. This sequence is essential in helping students build meaningful connections from direct experiences to more complex abstract understanding. Furthermore, Prosser and Bismarck (2023) the consistent use of concrete manipulatives in the early stages, combined with explicit instruction, significantly enhances students' retention of mathematical concepts and ability to explain procedures logically. Thus, CRA is a teaching strategy and a cognitive framework that integrates multisensory experiences into the mathematics learning process.

2.2 Joyful Learning

Joyful learning is an instructional approach that emphasises enjoyable, participatory, and emotionally motivating experiences. It focuses on creating an active, comfortable, and meaningful learning environment relevant to students' daily lives. This approach has been proven to enhance students' interest and academic achievement, including in mathematics learning (Ramadhani et al., 2024). Nabila et al. (2025) further support this by stating that joyful learning improves conceptual understanding, motivation, and students' active participation in mathematics classrooms.

Joyful learning is also a pedagogical approach that allows students to experience happiness while learning through meaningful and relevant activities. Rahmawati et al. (2024) explain that joyful learning places enjoyment at the heart of the learning process by incorporating games, stories, or interactive discussions to enhance motivation and build students' emotional connection with the subject matter. Their research shows that when elements of enjoyment are combined with a clear instructional structure, there is a significant increase in students' interest and self-confidence even in subjects often perceived as complicated, such as mathematics.

3. Method

3.1 Research Methods

This study employs a descriptive qualitative approach to describe students' mathematical representation abilities in learning fractions through the Concrete-Representational-Abstract (CRA) approach infused with joyful learning.

3.2 Research Subjects

This study's subjects were fourth-grade students from an elementary school in Cirebon. Three students from the entire group of participants, representing high, medium, and low

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mathematical ability categories, were selected as the primary participants for data collection. Research subjects were selected through a systematic series of stages, beginning with administering an initial test. Details of this process can be seen in **Figure 1**.

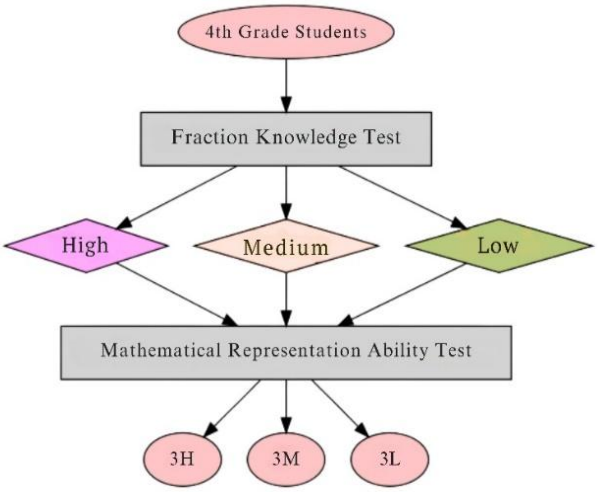

Figure 1. Research subject selection

Figure 1 illustrates the subject selection stages, which begin with administering a fraction concept understanding test to fourth-grade students. The test results are used to classify students into three ability categories: high, medium, and low. Subsequently, all students take a mathematical representation test to evaluate how they represent fractions in various forms, such as verbal, visual, and symbolic. Each category selects one student as a representative for further analysis. This process examines the relationship between students' initial understanding of fractions and their ability to represent the concept through the Concrete-Representational-Abstract approach infused with joyful learning.

3.3 Data Collection Techniques

Data collection in this study was carried out using two methods: written tests and semi-structured interviews. The written examinations assessed students' abilities in representing fraction concepts across the Concrete, Representational, and Abstract stages. Meanwhile, the interviews were conducted as a follow-up to the test results. They aimed to explore students' conceptual understanding of fractions more deeply and gain insights into their thought processes, problem-solving strategies, and the challenges they encountered during the game-based learning experience using the *Sirquid Game*.

3.4 Research Instruments

The instruments consisted of a mathematical representation test and an interview guide. The test items were designed in contextual form to evaluate students' abilities in verbally, visually, and symbolically representing fractions in accordance with the stages of the Concrete-Representational-Abstract (CRA) approach. The interview guide was developed to explore

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students' responses to the test, including their understanding of the material, their strategies while answering, and their impressions of the game-based learning process.

3.5 Data Analysis Techniques

Data analysis in this study refers to the Miles and Huberman model Sugiyono (2016), which includes three stages: data reduction, data display, and conclusion drawing or verification. The first stage, data reduction, involves sorting and categorising the students' mathematical representation test results based on ability levels (high, medium, and low) and marking responses corresponding to verbal, visual, and symbolic representation indicators. Interview data were also reduced to highlight students' understanding, thinking strategies, and impressions of learning through the game-based CRA approach. The second stage, data display, was conducted by organising the data into tables, interview experts, and descriptive narratives for each student category. This presentation allows a clear and systematic comparison of findings across representation types and ability groups. The third stage is conclusion drawing and verification, carried out to interpret the overall findings concerning the research objective, describing students' mathematical representation abilities in fraction learning based on the CRA approach.

3.6 Data Validity

The validity of the data in this study was ensured through several strategic steps aligned with qualitative research approaches. First, source triangulation was conducted by comparing data from students' written tests and interviews to verify the consistency of their understanding in representing fractions, particularly in verbal, visual, and symbolic representations. Second, method triangulation was applied by combining written test instruments to directly measure representation abilities with in-depth interviews exploring students' thought processes, problem-solving strategies, and impressions during game-based learning. Third, validation was carried out through member checking by reconfirming students' responses, as well as discussions with mathematics education experts to review the appropriateness of the instruments and data interpretations, ensuring their relevance and validity within the context of learning based on the Concrete-Representational-Abstract (CRA) approach infused with joyful learning.

4. Results

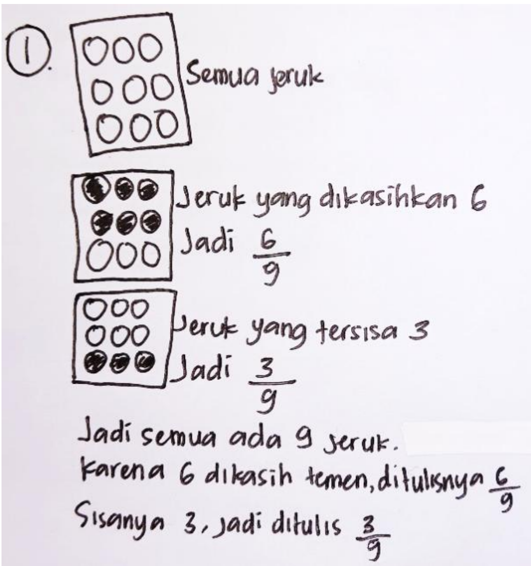
The research learning process was designed using a step-by-step approach through the Concrete-Representational-Abstract (CRA) strategy, packaged into mathematics play activities themed around the Sirquid Game. This game represents the implementation of joyful learning, emphasising cognitive aspects and integrating physical movement and enjoyment in learning fractions. Students were encouraged to participate actively by working on problems at their desks and through various physical activities such as running, jumping, and fast walking while completing fraction representation tasks.

The CRA stages began with the concrete activity, where students directly interacted with real-life situations that encouraged them to answer questions orally through the Run Speech

Fraction game, which simultaneously trained their verbal representation. In addition, in the representational stage, students were involved in the Jump and Draw Fraction game to arrange or select fraction images according to instructions, thereby supporting their visual understanding of fractions. Finally, in the abstract stage, students participated in the Walk Fast Fraction game to quickly write the symbolic form of fractions based on the understanding developed in the previous stages. This entire series of games-built fraction concepts gradually and created a fun, interactive learning atmosphere that motivated students to be more engaged.

The research results provide an overview of students' ability to represent fraction concepts after participating in learning using the Concrete-Representational-Abstract (CRA) approach combined with a joyful learning atmosphere. The analysis was conducted based on the mathematical representation indicators according to Vilegas (Amieny & Firmansyah, 2021), which classifies representations into three main categories: (1) verbal representation, which is the ability to explain concepts using sentences or words; (2) visual representation, which involves presenting concepts in the form of images, diagrams, or graphs; and (3) symbolic representation, which includes the use of mathematical symbols or models. To examine how these three forms of representation appeared in students' responses, the researcher provided contextual problems designed to explore students' ability to present fractions verbally, visually, and symbolically in an integrated manner.

The problem is as follows: "There are nine oranges in a box. You have given 6 of these oranges to your friends. Draw all the oranges and show which part has been given away and which remains. Write the part of the oranges given and the part remaining in the form of fractions, then explain in your own words why the fractions you made are accurate and correspond to the number of oranges."



Translate

All the oranges.
Oranges that were given away 6
So it's written as = $\frac{6}{9}$
Oranges that are left 3
So it's written as = $\frac{3}{9}$
So there are 9 oranges in total. Since 6 were given to friends, it's written as $\frac{6}{9}$
The remaining 3 are written as $\frac{3}{9}$

Figure 2. Answer of a student with high ability

The researcher analysed the students' written test results to assess how they applied verbal, visual, and symbolic representations in answer the given problem after participating in CRA-based learning packaged as a game. The analysis of these written test results is presented in **Figure 2**, which illustrates how a high-ability student represents fractions visually, symbolically, and verbally.

Based on the student's answers shown in **Figure 2**, the high-ability student demonstrates a comprehensive understanding in representing fractions visually, symbolically, and verbally. For the problem involving nine oranges, six given to friends, the student can draw all the oranges, shade the portion given away, and correctly write the fractions 6/9 and 3/9. The written explanation is coherent and logical, indicating that the student understands the relationship between the part and the whole. This ability reflects strong representation skills, linking the story context to visual and mathematical symbols, and verbally explaining the reasoning. The researcher conducts follow-up interviews to explore further the students' understanding of fraction representations demonstrated in the written answers. Below is the interview narrative with the high-ability student.

R (Researcher): *Earlier, you participated in the games Run Speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction. While playing, you had to speak, arrange fraction pictures, and write with numbers. Did you understand everything?*
S (Student): *"Yes, I understood everything! I like to say the answers directly, so arranging the fraction pictures was easy because I already understood. I could also write with numbers because I know how to calculate."*
R: *Before you answered those questions, what did you usually think about first to figure out the fractions?*
S: *"I first think about how many there are in total, then how many parts are being asked. Once I know that, I could say, arrange, or write the fractions."*
R: *From those activities, do you think speaking directly, arranging pictures, and writing numbers helped you understand fractions better?*
S: *"Definitely! Because I could imagine the fractions from the pictures, explaining them is easy, and I could also write them."*
R: *Did you find the games fun? Do you like learning fractions with games like that?*
S: *"Very fun! I like everything. So, I'm excited to learn fractions because it feels like playing and thinking simultaneously!"*

Based on the interview results, the high-ability student demonstrated strong mastery of verbal, visual, and symbolic representations in learning fractions through the CRA approach based on the Sirquid Game. Verbally, the student could correctly state the fractions after considering the whole and the parts being asked, and felt that speaking directly helped deepen their understanding of the concept. In the visual representation, the student could accurately arrange fraction images through the Jump and Draw Fraction game, which they found enjoyable and helpful for concretely imagining the fractions. The student understood the numerator and denominator well in the symbolic representation and wrote the fractions based on a coherent thought process. They also considered the walk fast fraction game fun because it was challenging and made them think quickly while physically active.

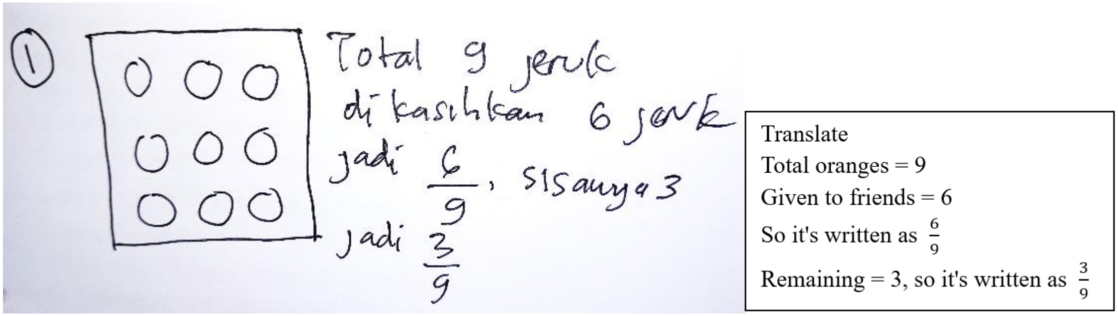
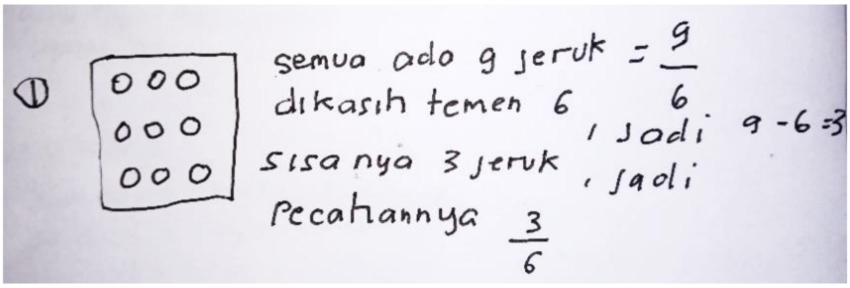


Figure 3. Answer of student with moderate ability

Accordingly, based on the student's answers shown in **Figure 3**, the moderate-ability student demonstrates a pretty good basic understanding of representing fractions, particularly in symbolic form. In the question about dividing the oranges, the student writes the fractions 6/9 for the part given to friends and 3/9 for the remaining part. Although the drawing does not clearly show the difference between the given and remaining parts, the written explanation shows that the student understands the relationship between the number of parts and the whole. This indicates that the student can connect verbal information with symbolic representation, but still requires guidance to strengthen the visual representation more meaningfully. The researcher conducts a follow-up interview to further explore the students' understanding of fraction representations in the written answers. The following is the interview narrative with a student of moderate ability.

R (Researcher): *You joined the Run speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction games earlier. You had to say your answers, arrange fraction pictures, and write fractions with numbers. Did you understand all the games?*
S (Student): *"I understand, but sometimes I get confused. When I say the answers, I think first about which part is being asked, then about how much of the whole it is. I made some mistakes when arranging the pictures, but finally got it right. Writing the numbers is hard; I often forget which goes on top and which is on the bottom. But playing the games helped me understand fractions more because I could simultaneously look, speak, and write."*
R: *Did these activities help you understand fractions better?*
S: *"Yes, even though I sometimes still have to ask the teacher, I know that fractions can be understood through pictures, speaking, and writing numbers."*
R: *What do you think about the games? Did you enjoy learning like that?*
S: *"Yes! I like the jumping and fast walking parts. So, the learning isn't boring, it's more like playing."*

Based on the interview results, the moderately able student demonstrates an adequate understanding of verbal, visual, and symbolic representations, although it is not yet fully stable. Verbally, the student is able to respond, although somewhat hesitantly, and feels that expressing answers orally helped in understanding fractions. Regarding visual representation, the student recognised that drawings represent parts of a whole and described arranging fraction visuals as enjoyable, despite initially making mistakes. For symbolic representation, the student began to understand how to write fractions based on the shaded and total parts, though still confused about distinguishing the numerator and denominator. Movement-based games, such as fast walking, were also perceived as fun and motivated the student to participate, even though teacher guidance was still needed.



Translate
There are 9 oranges in total = $\frac{9}{6}$
6 were given to friends
So, 9 - 6 = 3 left
There are 3 oranges left
The fraction is $\frac{3}{6}$

Figure 4. Answer of a student with low ability

Additionally, based on the student's answer in **Figure 4**, the student with low ability demonstrates a limited understanding of connecting verbal information with visual and symbolic representations. Although the student can draw nine whole oranges, explaining which parts are given away and which remain is still inaccurate. The student miswrites the fraction as 3⁄6, instead of 6⁄9 for the part given away and 3⁄9 for the remaining part. This indicates that the student has not yet fully grasped the concepts of numerator and denominator, and cannot accurately translate a contextual situation into fractional form. Furthermore, the visual representation does not clearly distinguish between the given and remaining parts, making the meaning of the fraction less apparent. These findings highlight the need for more intensive guidance to develop fractional representation understanding in lower-ability students. The researcher conducts a follow-up interview to explore the student's understanding of fraction representation shown in the written response. The following is the interview transcript with the low-ability student.

R (Researcher): *Earlier, you participated in the Run Speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction games. In those games, you had to say the answers, arrange fraction pictures, and write the fractions using numbers. Did you understand all the games?*
S (Student): *"I didn't understand everything. Sometimes, I get confused about what to say when asked about fractions. I tried to think about which part was being asked, but I often forgot. Arranging the pictures was also hard; I kept choosing the wrong ones. Writing the fraction numbers was the hardest; I still don't know what the top number means. But I'm starting to understand that fractions are like parts of a whole. I liked the jumping and fast walking parts the most because they were fun, but I was also afraid of giving the wrong answer."*
R: *Did today's activity help you understand fractions better?*
S: *"Not fully understood yet, but I'm starting to understand that fractions are parts of a picture."*
R: *Do you think the game was fun? Do you enjoy learning that way?*
S: *"Yes, it was fun! Because I could move around and not just sit in class. But I'm still confused when it comes to answering."*

Based on the interview results, the student with low ability demonstrated a limited understanding of fraction representation. Verbally, the student had difficulty expressing answers and was confused about which parts to describe, although speaking about fractions helped a

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little. In the visual representation, the student could not accurately construct fraction images or grasp that a fraction is parts of a whole. Nevertheless, the student remained enthusiastic during the physical activities involved in the game, such as jumping, even though arranging the pictures was still confusing. In the symbolic representation, the student did not understand numerators and denominators and tended to guess. The student expressed enjoyment during the Walk Fast Fraction game due to the physical movement involved, but also felt anxious when required to write answers for fear of being wrong.

Students with high ability demonstrated a comprehensive and consistent mastery in representing fractions verbally, visually, and symbolically. They could explain their thought processes logically and connect contextual information and the appropriate fraction form. Students with moderate ability showed a fair understanding, particularly in symbolic representation, although they still experienced confusion when linking visual and verbal representations simultaneously, especially in complex problem contexts. Meanwhile, students with low ability had difficulty distinguishing between numerators and denominators and could not accurately illustrate or write fractions in symbolic form. Nevertheless, students across all three ability levels showed enthusiasm for learning through games.

Table 1. Data triangulation

Student Category	Verbal Representation	Visual Representation	Symbolic Representation	Interview/ Observation
High	The student is able to explain the concept of fractions accurately and coherently, with a deep understanding of the relationship between parts and the whole.	The student draws fraction objects with accurate detail, visually representing the partitioning of parts according to the problem's context.	The student writes the fractions (6/9 and 3/9) correctly and in accordance with the problem's context.	The student demonstrates a good understanding of the material, feels motivated, and enjoys the learning process through engaging games.
Moderate	Students demonstrate a basic understanding of fraction concepts, but still show uncertainty when explaining verbally.	The fraction drawings made by the student show unclear division of parts, indicating the need for further guidance to improve visual understanding.	The student writes the fractions correctly, but still shows confusion in determining the positions of the numerator and denominator.	The student admits feeling helped by the learning method, although still needing teacher guidance, and enjoys the game as a learning medium.
Low	The student experiences difficulty in explaining the fraction concept verbally and is less accurate in identifying the fraction parts.	The drawing does not accurately represent the fraction division, indicating limited visual understanding.	Students often miswrite fractions (e.g., 3/6 instead of 6/9), indicating a lack of mastery of the symbolic fraction concept.	Students express a lack of understanding of the material, but still show enthusiasm for the game's physical activity aspects.

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With a joyful learning atmosphere, the physical activities integrated into the CRA approach successfully created an enjoyable learning environment and encouraged active student engagement. This indicates that the approach supports cognitive development and positively impacts students' affective aspects. However, guidance is still necessary, especially for students with moderate and low abilities, to help them build conceptual understanding more independently and meaningfully.

Based on the results of data triangulation through observation, written tests, and interviews, **Table 1** presents a detailed description of students' tendencies in representing the concept of fractions in verbal, visual, and symbolic forms. This description illustrates the students' thought processes during fraction learning using the Concrete-Representational-Abstract (CRA) approach within a joyful learning environment.

5. Discussion

The Concrete-Representational-Abstract (CRA) approach, combined with a joyful learning atmosphere, has proven effective in gradually developing students' mathematical representation abilities. CRA offers a learning sequence that begins with concrete experiences, followed by visual representations through drawings, and ultimately, the use of abstract mathematical symbols (Amieny & Firmansyah, 2021). This approach facilitates deep conceptual understanding as it aligns with children's cognitive development stages. In line with this, Toong (2020) emphasises that mathematical ideas are inherently abstract as they are formed in the mind; therefore, to aid comprehension, these concepts need to be bridged through concrete representations that can be seen or touched. These concrete representations help students grasp the essence of mathematical ideas before they are able to process them symbolically.

Joyful learning is an instructional approach that emphasises using structured and enjoyable methods to create a harmonious and supportive learning environment (Khusniyah, 2018). In this study, the Concrete-Representational-Abstract (CRA) approach was implemented within a joyful learning atmosphere to create a meaningful and enjoyable learning experience for students. The application was done through three educational games: Run Speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction, each designed to gradually train students' verbal, visual, and symbolic representation skills. This approach is believed to effectively support the active and progressive development of students' conceptual understanding in mathematics. In line with this, Zulfakri et al. (2019) affirm that CRA has the potential to enhance students' mathematical representation and problem-solving abilities.

These findings align with Suwarsih (2018), who stated that learning in an enjoyable atmosphere can lead to more optimal learning outcomes. Suripatty et al. (2019) also added that learning through play combines exploration, discovery, experimentation, and creativity in a joyful environment, thereby enhancing student engagement and understanding. High-ability students consistently mastered the three forms of representation: verbal, visual, and symbolic. They could explain fractions orally, depict them accurately, and write them in appropriate mathematical symbols. Interviews also revealed that the CRA approach, packaged as games, strengthened conceptual understanding and increased students' motivation and enthusiasm for learning. This is supported by Sufiani and Marzuki (2021), who stated that joyful learning can boost students' interest and enthusiasm by engaging them in fun activities that foster creativity in a positive atmosphere.

Students with moderate ability demonstrated a good understanding, particularly in symbolic representation. However, they still encountered confusion when constructing fraction diagrams and integrating verbal and visual information simultaneously. Nevertheless, the enjoyable learning atmosphere helped them remain focused and motivated. The enthusiasm that emerged during the learning process played an essential role in enhancing learning motivation and encouraging students' potential to understand basic concepts more optimally (Arifa & Utaminingtyas, 2024). Meanwhile, students with low ability faced greater challenges, such as difficulty distinguishing between the numerator and denominator and an inability to represent fractions contextually. Although they often felt anxious when solving problems, their interest in the game-based activities remained high. This indicates that a positive and enjoyable learning environment plays a significant role in creating meaningful learning experiences (Hurriyati et al., 2022).

This reinforces the essentials of a positive learning atmosphere in supporting students' understanding of mathematical concepts. These findings align with those of Sardin et al. (2022), who found that joyful learning using card-based media can enhance students' mathematical understanding. The difference lies in the approach used, as this study integrates the CRA model to guide students progressively from the concrete stage to the abstract stage while maintaining an enjoyable learning environment. Thus, the learning process increases active participation and deepens students' conceptual thinking.

In addition, the study by Putri and Wulandari (2024) further supports these findings by showing that a joyful learning environment has a significant impact on improving students' mathematical abilities. They found that implementing joyful learning and ice-breaking activities encouraged active participation and fostered interest in learning mathematics. These activities created a more relaxed and supportive classroom atmosphere, making students more emotionally and cognitively prepared to absorb learning material. Ultimately, this condition positively affected students' concentration, engagement, and learning outcomes.

The learning approach based on the Concrete-Representational-Abstract (CRA) model, combined with a joyful learning atmosphere, offers more comprehensive advantages. Not only does it create an enjoyable learning environment, but it also provides a systematic instructional structure through the concrete, representational, and abstract stages. Through these stages, students feel comfortable during the learning process and gain opportunities to gradually and deeply develop their conceptual understanding. This is supported by Milton et al. (2023), who assert that integrating the CRA approach in mathematics learning can significantly enhance students' conceptual understanding and self-confidence.

Implementing the Concrete–Representational–Abstract (CRA) approach in a joyful learning environment has improved students' mathematical representation abilities. This step-by-step approach, starting from concrete experiences, followed by visual representations, and culminating in abstract understanding, allows students to build a more profound, more guided, and enjoyable sense of fraction concepts. Therefore, teachers must carefully design learning activities and consider diverse learning needs to ensure this approach can be optimally applied in various classroom situations.

6. Conclusion and Implications

Implementing the Concrete–Representational–Abstract (CRA) approach combined with a joyful learning atmosphere has proven to be an effective teaching strategy in developing elementary students' mathematical representation abilities, particularly in fractions. Through the concrete, representational, and abstract stages packaged in games such as Run Speech Fraction, Jump and Draw Fraction, and Walk Fast Fraction, students' verbal, visual, and symbolic representation abilities are effectively trained within meaningful contexts. Learning presented in games creates an enjoyable learning environment, encourages active engagement, reduces anxiety toward mathematics, and increases students' confidence in expressing mathematical ideas.

The best practice demonstrated in this study is the step-by-step CRA approach designed in a physical-interactive game format, effectively enhancing students' engagement, motivation, and confidence throughout the learning process. Students not only understand fractions through symbols or pictures but are also able to explain their meanings verbally based on context. This strategy accommodates various levels of student ability from high to low while maintaining a joyful and non-pressuring learning environment.

The implication is that teachers can adopt this approach by integrating simple physical activities and educational game media into mathematics learning, especially for abstract topics like fractions. Such an approach facilitates gradual understanding and encourages comprehensive student participation and motivation. Learning designed with attention to students' cognitive and emotional development is more likely to create meaningful and memorable learning experiences.

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